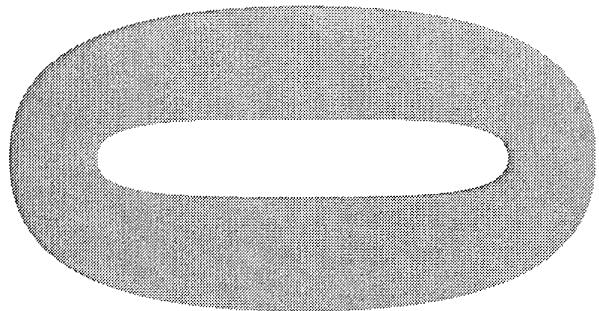




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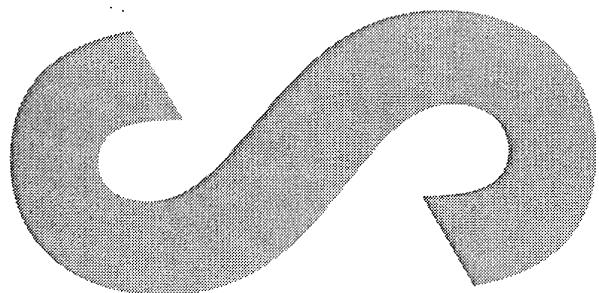
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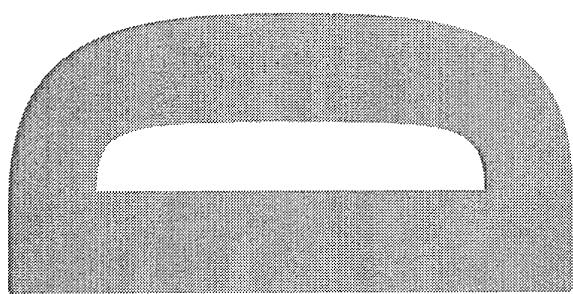
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# Review of Improved Methods for Analysing Load Attraction and Thermal Effects in Bonded Composite Repair Design

*A.B. Harman and K.F. Walker*

**Air Vehicles Division**  
Platforms Sciences Laboratory

DSTO-TR-1546

## ABSTRACT

Adhesive bonded repairs to aircraft involving metallic and composite structures have proven to be an effective, efficient means of repair and life extension. The simplified closed form equations used by the RAAF in an Engineering Standard (DEF(AUST)9005) have proven to be effective and conservative. Recent work, however, has identified improved equations to account for load attraction into the stiffened repaired area, and evaluate the thermally induced stresses in the repaired structure and the patch. The improved equations were compared with the current methods, using the repair to a 2024-T851 aluminium alloy F-111 lower wing skin with a boron epoxy repair patch, bonded with FM-73 film adhesive. The improved methods will reduce the unnecessary conservatism inherent in DEF(AUST)9005 and therefore allow some repairs to proceed where they may otherwise have been rejected. Repairs will also be designed to operate more efficiently.

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# Review of Improved Methods for Analysing Load Attraction and Thermal Effects in Bonded Composite Repair Design

## Executive Summary

Adhesive bonded repairs to aircraft involving metallic and composite structures have proven to be an effective, efficient means of repair and life extension. The simplified closed form equations used by the RAAF in an Engineering Standard (DEF(AUST)9005) have proven to be effective and conservative.

Recent work, however, has identified improved equations to account for load attraction into the stiffened repaired area, and evaluate the thermally induced stresses in the repaired structure and the patch. The improved equations were compared with the current methods using the repair to a 2024-T851 Aluminium Alloy F-111 lower wing skin with a boron epoxy repair patch bonded with FM-73 film adhesive as an example.

The improved equations produce small increases in the stress in the skin under the patch, the adhesive strain and the repaired stress intensity factor. However, the stress in the skin at the edge of the patch and the stress in the patch itself are considerably reduced. By removing unnecessary conservatism in DEF(AUST)9005, repairs to RAAF aircraft may now be able to safely proceed whereas previously they could have been rejected. The improved methods presented in this report will allow an increase in the scope of application of adhesive bonded repair methods. This will increase the availability and extend the structural life of RAAF aircraft.

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## 1. Introduction

Adhesive bonded repairs to aircraft involving metallic and composite structures have proven to be an effective means of repair and life extension (References 1-4). In Australia, the procedures for designing such repairs have been documented in References 5 and 6. The design equations have been derived from a number of sources. The main sources are References 1 and 7.

The equations are in closed form and are relatively simple to apply. Despite their simplicity, they have proven to be effective for real life practical examples, including a repair to primary aircraft structure in one case (References 3 and 4).

The equations are, however, restricted basically to cases of either a double sided symmetric repair, or single sided repair where full out of plane support or restraint is provided by the sub-structure (eg a spar). Although the equations as currently contained in References 5 and 6 have proven to work well, it has recently become evident that improvements are possible in the following areas;

1. Calculating the load attraction factor, and
2. Accounting for the thermally induced stresses in the repaired structure and patch.

The improvements are drawn from References 8 and 9, and are clearly identified and articulated in this report. The final results in terms of stresses and stress intensity factors do not change significantly as a result. However, the revised equations are considered superior and recommended for use in future versions of References 5 and 6.

## 2. Current and Improved Methods

### 2.1 Load Attraction

#### 2.1.1 Current Method

The load attraction factor,  $\Omega_L$ , allows for load attraction into the repaired area due to stiffness added by the patch. Currently, the guidance given in Reference 6 is as follows:

"Typical values are  $\Omega_L = 1.2$  for a square patch to 2.0 for a patch which is very long in the major load direction"

### 2.1.2 Improved Method

Using the equations detailed in Section 7.4 of Reference 8, it is possible to estimate  $\Omega_L$  based on the patch width and length. The procedure is as follows:

Patch Shape Ratio,  $Shape = W_h / L_h$

$$\text{Stiffness Ratio : } s = \frac{E_o t_o}{E_i t_i}$$

Where:

$W_h$ =	Half Patch Width
$L_h$ =	Half Patch Length
$E_i$ =	Youngs Modulus of Skin or Plate
$t_i$ =	Thickness of Skin or Plate
$E_o$ =	Youngs Modulus of Patch
$t_o$ =	Thickness of Patch

From Reference 8, Equation 7.18,

$$z = 3(1 + s)^2 + 2(1 + s)[(1 / Shape) + Shape + \nu s] + 1 - \nu^2 s^2$$

Where:

$\nu$  = Poisson's Ratio of Skin

$$\text{Stress reduction factor, } \Phi = \frac{1}{z} [4 + 2(\frac{1}{Shape}) + 2Shape + s(3 + \nu + \frac{2}{Shape})]$$

From Reference 8, Equation 7.20

$$\text{Load attraction factor, } \Omega_L = (1 + s)\Phi$$

These equations are for the simplified case of uniaxial loading and where the plate or skin and the patch are both isotropic and have the same poisson's ratio. In fact, the skin or plate is normally isotropic, but the patch is often orthotropic. The shape of the patch is also assumed to be semi elliptic. Nevertheless, the equations presented will provide a close approximation for the load attraction factor. For the case of a patch with a shape ratio of one, the load attraction factor is approximately 1.2.

## 2.2 Thermal Effects

### 2.2.1 Current Method

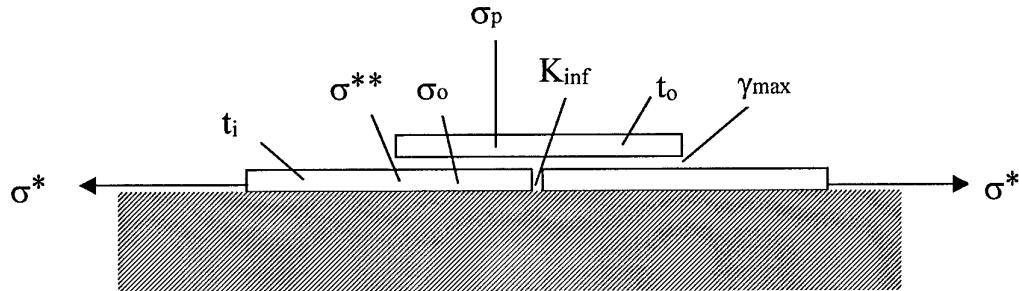


Figure 1 Relevant Parameters for Single-Sided, Fully Supported Bonded Repair Design Analysis

The main quantities to be calculated are as follows (see Figure 1):

- $\sigma^*$  : Far field applied stress
- $\sigma^{**}$  : Stress in the skin or plate at the edge of the patch
- $\sigma_o$  : Stress in the skin or plate under the repair (ignoring the crack)
- $\sigma_p$  : Stress in the patch
- $K_{inf}$  : Repaired stress intensity factor
- $\gamma_{max}$  : Maximum shear strain in the adhesive

The equations for  $\sigma^{**}$  and  $\sigma_o$  are updated with the improved method, and as such the existing formulae within Reference 5 and 6 are presented below. With regard to the other related formulae, the equations for  $K_{inf}$ , and  $\gamma_{max}$  do not change and remain as per References 5 and 6. The equation for  $\sigma_p$  does not change if there is a crack present. If there is no crack in the parent material under the patch, such as in the case of a composite reinforcement doubler, the equation for  $\sigma_p$  changes.

#### Existing equation for $\sigma^{**}$ :

The equation given in References 5 and 6 is as follows:

$$\sigma^{**} = \Omega_L \sigma^* + E_i [(\alpha_o - \alpha_{i_{eff}})(RT - T_{CURE}) + (\alpha_o - \alpha_i)(T_{OP} - RT)]$$

Where:

- $\Omega_L$  = Load Attraction Factor
- $\sigma^*$  = Remote Applied Stresses

$\alpha_o$ =	Coefficient of Thermal Expansion (CTE) of the Patch
$\alpha_{ieff}$ =	Effective CTE of the Skin/Plate accounting for Constraint of Surrounding Unheated Structure
RT =	Room Temperature (75°F)
T <sub>CURE</sub> =	Cure Temperature
$\alpha_i$ =	CTE of the Skin/Plate
T <sub>OP</sub> =	Operating Temperature

The concept is that during cooling the structure from the cure temperature back to ambient temperature, the effective skin or plate CTE will be less than the full material value. This is due to the constraint provided by the surrounding unheated structure (i.e. the localised nature of heating for cure).

The effective CTE is calculated using the following simple formula (References 5 and 6):

$$\alpha_{ieff} = \frac{\alpha_i(\nu+1)}{2}$$

This effective CTE is applied only to the temperature change from cure to ambient. The change from ambient to operating temperature affects the whole structure (eg. an aircraft flying to altitude) so the full CTE is applied.

#### Existing equation for $\sigma_o$ :

The equation given in Reference 6 is as follows:

$$\sigma_o = \frac{\sigma^{**}(E_i t_i)}{E_o t_o + E_i t_i}$$

Or

$$\sigma_o = \frac{\sigma^{**}}{1+s}$$

#### Existing equation for $\sigma_p$ :

In cases where a crack is present, the equation given in Reference 6, Appendix 2 to Annex C, Chapter 6 equation (6.C.2.18), is as follows:

$$\sigma_p = \frac{t_i}{t_o} \sigma^{**}$$

For cases where there is no crack, the equation given in Reference 6, Appendix 4 to Annex C, Chapter 6 equation (6.C.4.16), is as follows:

$$\sigma_p = \frac{E_o t_i}{E_o t_o + E_i t_i} \sigma^{**}$$

### 2.2.2 Improved Method

The approach for the improved method is taken from References 8 and 9. The concept is that the repair application process is broken down into individual stages to account for effects due to temperature changes. The stages are as follows:

- Stage 1 : Localised heating to cure the adhesive
- Stage 2 : Localised cooling from the cure temperature to ambient
- Stage 3 : Total heating or cooling to the "operating" temperature at which the repair is evaluated

The applied stress is added to the thermal stresses.

As discussed in References 8 and 9, the closed form equations are greatly simplified to enable analytic derivation. The primary simplifying assumption is that the parent/skin/plate and patch/reinforcement are made from isotropic material. In the typical repair case, the structure being repaired is metallic and is approximately isotropic. However, the repair patch is often made from non-isotropic composite laminae.

It is demonstrated in Reference 9 that the effect of this assumption on the stress predictions in the patch and plate is insignificant. If the repair designer assumes the major or longitudinal properties of the orthotropic patch apply as its isotropic properties, then the results for the skin/plate stresses are reasonable and conservative in the primary load direction.

Another assumption made is that the structure surrounding the repair patch is infinite and only the area of the repair patch itself is heated. This is also considered to be a reasonable approximation for the typical repair situation in a real aircraft repair. In other words, the area of the repair is small compared to the overall dimensions of the structure.

#### Revised/improved equation for $\sigma^{**}$ :

The component due to the applied stress is given by:

$$\sigma^{**}_{\text{applied}} = \Omega_L \sigma^*$$

The component due to heating (Stage 1) is given by Reference 9, Equation 3:

$$\sigma_{heating}^{**} = \frac{-\alpha_i E_i (T_{CURE} - RT)}{2}$$

The component due to cooling to ambient (Stage 2) is given by Reference 8, Equation 7.102:

$$\sigma_{cooling}^{**} = -\alpha_i E_i (RT - T_{CURE}) \frac{[1 - \nu_{12o} + s(1 - \nu) \frac{\alpha_o}{\alpha_i}]}{[2(1 - \nu_{12o}) + (1 - \nu^2)s]}$$

Where:

$\nu_{12o}$  = Longitudinal Poisson's Ratio of the Patch

The component due to heating or cooling the whole structure to the operating temperature (Stage 3) is given by Reference 8, Equation 7.104:

$$\sigma_{operating}^{**} = -\alpha_i E_i (T_{OP} - RT) \frac{[(1 - \nu)(1 - \frac{\alpha_o}{\alpha_i})s]}{[2(1 - \nu_{12o}) + (1 - \nu^2)s]}$$

The total is given by:

$$\sigma^{**} = \sigma_{applied}^{**} + \sigma_{heating}^{**} + \sigma_{cooling}^{**} + \sigma_{operating}^{**}$$

Revised/improved equation for  $\sigma_o$ , stress in the skin/plate under the patch ignoring the crack<sup>1</sup>:

The component due to the applied stress is given by:

$$\sigma_{o\_applied} = \frac{(\Omega_L \sigma^*)}{1 + s}$$

The component due to heating (Stage 1) is given by Reference 9, Equation 3:

$$\sigma_{o\_heating} = \frac{-\alpha_i E_i (T_{CURE} - RT)}{2}$$

---

<sup>1</sup> Note that the equations given in Reference 6 for calculating the repaired stress intensity rely on a value of  $\sigma_o$  in the absence of the crack

The component due to cooling to ambient (Stage 2) is given by Reference 8, Equation 7.99:

$$\sigma_{o\_cooling} = -\alpha_i E_i (RT - T_{CURE}) \left[ \frac{\left[1 - \nu_{12o} + \left(1 - \frac{\alpha_o}{\alpha_i}\right)(1 + \nu)s\right]}{\left[2(1 - \nu_{12o}) + (1 - \nu^2)s\right]} \right]$$

The component due to further cooling the whole structure to the operating temperature (Stage 3) is given by Reference 8, Equation 7.104:

$$\sigma_{o\_operating} = -\alpha_i E_i (T_{OP} - RT) \left[ \frac{\left[(1 + \nu)\left(1 - \frac{\alpha_o}{\alpha_i}\right)s\right]}{\left[2(1 - \nu_{12o}) + (1 - \nu^2)s\right]} \right]$$

The total is given by:

$$\sigma_o = \sigma_{o\_applied} + \sigma_{o\_heating} + \sigma_{o\_cooling} + \sigma_{o\_operating}$$

#### Revised/improved equation for $\sigma_p$ :

In the case where a crack is present, the equation given in Reference 6 for stress in the patch is reasonable. It is based on the concept of load compatibility, i.e. the load transmitted at the edge of the patch must be transmitted fully through the patch at the crack location.

However, for situations where we want to obtain the stress in the patch in the absence of a crack, the equation in Reference 6 is not correct. In that situation, all load transferring into the patch is assumed to shed from the parent material, and pass through the patch over the defect. As such, it can be seen in Reference 6 that the following formulae apply:

The applied stress is given by:

$$\sigma_{p\_applied} = \frac{E_o t_i}{E_o t_o + E_i t_i} \sigma_{applied}^{**}$$

Or

$$\sigma_{p\_applied} = \frac{s t_i}{(1 + s) t_o} \sigma_{applied}^{**}$$

Note that this equation is basically the same as the existing equation in Reference 6, and it is based on load and strain compatibility. However, this is only for the applied stress. The heating, cooling and operating components must be added separately.

Stress due to heating (Stage 1) is zero because the patch is free to expand, i.e.:

$$\sigma_{p\_heating} = 0$$

Stress due to cooling to ambient (Stage 2) is given by Reference 8, Equation 7.103:

$$\sigma_{p\_cooling} = \alpha_i E_o (RT - T_{CURE}) \left[ \frac{(1 + \nu - 2 \frac{\alpha_o}{\alpha_i})}{[2(1 - \nu_{12o}) + (1 - \nu^2)s]} \right]$$

The stress due to heating or cooling the whole structure to the operating temperature (Stage 3) is given by Reference 8, Equation 7.105:

$$\sigma_{p\_operating} = \alpha_i E_o (T_{OP} - T_{CURE}) \left[ \frac{2(1 - \frac{\alpha_o}{\alpha_i})}{[2(1 - \nu_{12o}) + (1 - \nu^2)s]} \right]$$

The total is given by:

$$\sigma_p = \sigma_{p\_applied} + \sigma_{p\_heating} + \sigma_{p\_cooling} + \sigma_{p\_operating}$$

### 3. Example Case Based on F-111C Lower Wing Skin Repair

The revised formulae have been compared with the current method for an example case based on the F-111C lower wing skin repair (References 3 and 4). The case involved the application of a 14 layer boron epoxy (5521/4) patch bonded with FM 73 film adhesive cured under positive pressure at 180°F.

The patch only needed 10 layers of unidirectional boron epoxy oriented in the 0° direction. The other 4 layers were orientated at ± 45° to the primary load direction. For the purposes of simplifying the comparison, a 10 layer unidirectional patch was assumed for repair design. As such, the extensional stiffness (Et) of the patch and skin approximately matched.

The wing skin is manufactured from 2024-T851 aluminium alloy. The repair was evaluated at operating temperatures of -40, 75 and 167°F. The remote applied stress is

37,100 psi (37.1 ksi). Results are provided in this section and mathematical workings are provided in Appendix A.

### 3.1 Material Properties

The relevant material properties are as follows:

#### 3.1.1 FM-73 Structural Film Adhesive

FM-73 Structural film adhesive properties were obtained from Reference 6 and are detailed in Table 1 below.

*Table 1 FM-73 Structural Film Adhesive Properties*

Parameter	Values		
	-40°F	RT (75°F)	167°F
$\tau_p$ , Shear Stress at Linear Limit (psi)	8230	6052	3534
$G$ , Shear Modulus (psi)	115306	73323	13522
$\gamma_{\max}$ , Shear Strain at Failure	0.1915	0.5774	0.8276
$\gamma_e$ , Elastic Shear Strain Limit	0.0723	0.0804	0.5896
$\gamma_p$ , Plastic Shear Strain Limit	0.1192	0.497	0.2380
$\eta$ , Adhesive Thickness (inches)	0.013"	0.013"	0.013"

#### 3.1.2 2024-T851 Aluminium Alloy

The properties of 2024-T851 aluminium alloy are as per Reference 10 and are detailed in Table 2 below:

*Table 2 2024-T851 Aluminium Alloy Properties*

Parameter	Value
$t_i$ , Skin Thickness (inches)	0.14"
$\sigma_{yi}$ , Yield Stress (psi)	59000
$\sigma_{ui}$ , Ultimate Stress (psi)	65000
$K_c$ , Fracture toughness ( $\text{psi}\sqrt{\text{inch}}$ )	42000
$E_i$ , Youngs Modulus (psi)	$10.5 \times 10^6$
$v$ , Poisson's Ratio	0.31
$\alpha_i$ , CTE (in/in/°F)	$12.6 \times 10^{-6}$

#### 3.1.3 Boron Epoxy 5521/4

The properties of Boron Epoxy 5521/4 are as per Reference 6 and are detailed in Table 3 below:

Table 3 Boron Epoxy 5521/4 Properties

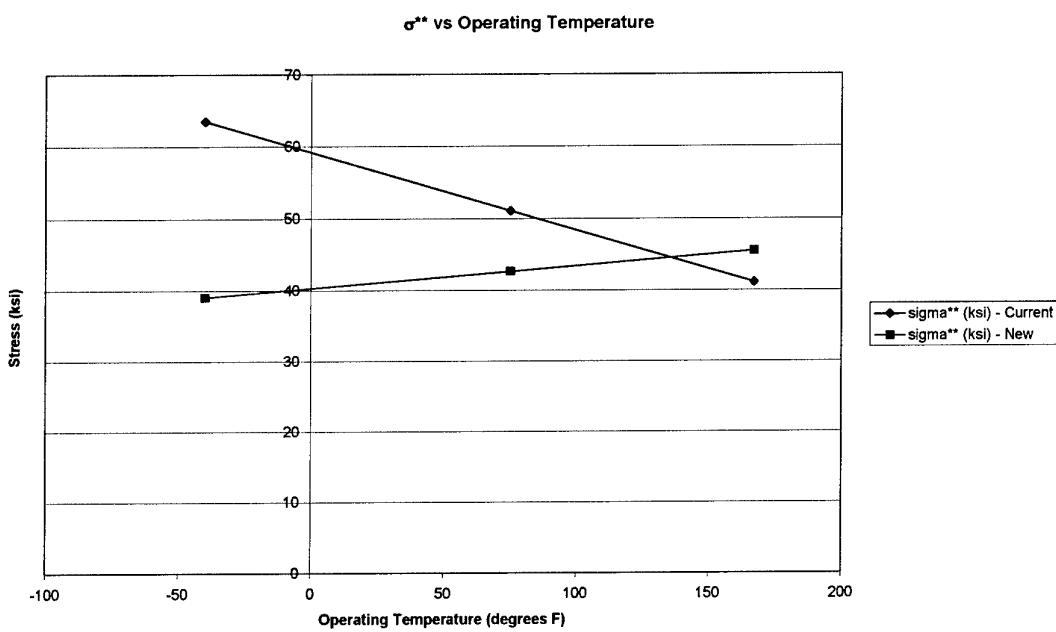
Parameter	Value
$\epsilon_o$ , Ultimate longitudinal strain	0.00655
$E_o$ , Youngs Modulus (psi)	$30 \times 10^6$
$\alpha_o$ , CTE (in/in/°F)	$2.3 \times 10^{-6}$

### 3.2 Results

The equations detailed in Section 2 were assembled in MATHCAD worksheets provided in Appendix A. For comparison purposes, the load attraction factor,  $\Omega_L$ , was set to a value of 1.2. The key results are provided in Table 4 below.

Table 4 Comparison of results

Parameter	DEF(AUST)9005 (current)			Proposed Equations		
	-40°F	RT (75°F)	167°F	-40°F	RT (75°F)	167°F
$\sigma^{**}$ (ksi)	63.52	51.08	41.13	39.04	42.63	45.50
$\sigma_o$ (ksi)	30.82	24.78	19.96	32.00	25.19	19.74
$\gamma_{max}$	0.151	0.142	0.242	0.160	0.146	0.240
$K_{inf}$ (ksi $\sqrt{inch}$ )	23.81	21.16	25.69	24.84	21.54	25.41
$\sigma_p$ (ksi) (cracked)	171.0	137.50	110.70	105.10	114.80	122.50
$\sigma_p$ (ksi) (no crack)	88.05	70.81	57.02	18.95	46.94	69.33

Figure 2  $\sigma^{**}$ , Stress in the Skin/Plate at the End of the Patch vs Operating Temperature

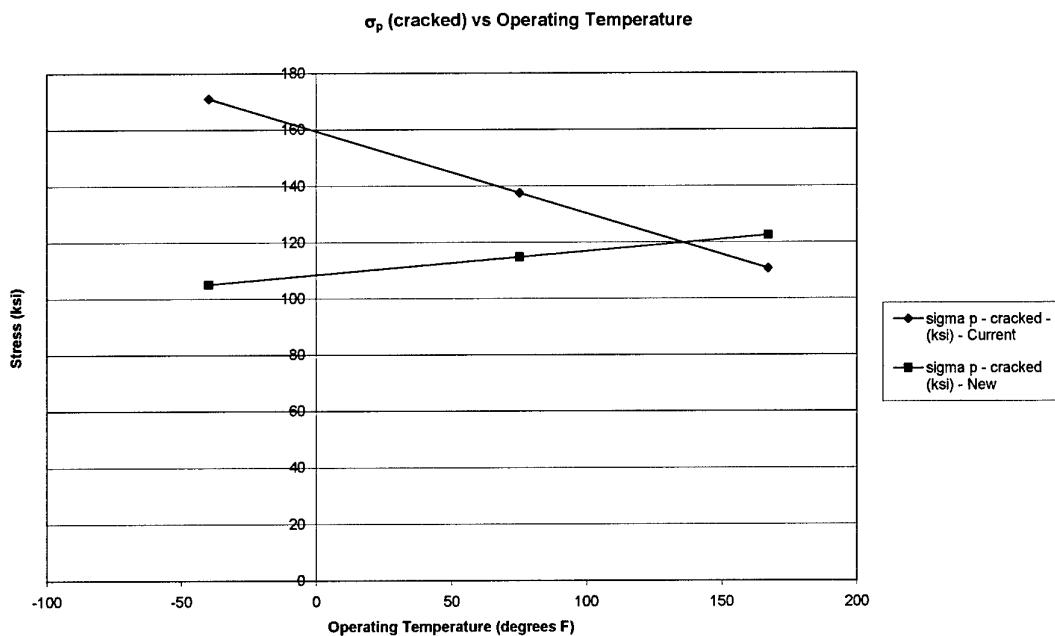


Figure 3  $\sigma_p$ , Stress in the Patch (with crack in the skin) vs Operating Temperature

#### 4. Discussion

From the results provided in Table 4 and shown in Figure 2, it is clear that the proposed procedure will result in significantly reduced values of  $\sigma^{**}$ , the stress in the skin or plate at the end of the patch. The exception to this is the high temperature (167 °F) case where the proposed procedure gives 45.5 ksi compared with 41.3 ksi from the current equation.

A more important observation however comes from a close examination of the trend of stress level as a function of operating temperature. The improved equations capture the trend expected for a physical problem of this type, whereas the existing equations give a contradictory result. In the case of the current equations, the stress decreases as operating temperature increases. The reverse is true for the proposed equations.

An explanation, in physical terms, is offered as follows. The stress due to the applied load, the heating stage, and the cooling to ambient is consistent for each case. The difference comes in during the stage from ambient to the operating condition. When you cool the entire structure from 75 °F down to -40 °F, it makes sense that the surrounding structure will want to contract more than the inclusion (the patch/skin combination is referred to as an inclusion). This is due to the inclusion having a lower

CTE than the skin/structure. Displacement compatibility dictates that the inclusion is forced to fit into a smaller hole. As such, compressive stresses develop. The reverse is true when you move to a higher than ambient operating temperature. The existing equations do not capture this physical trend.

As seen in Table 4 and Figure 3, the stress in the patch for the cracked case is also significantly less for the proposed equations apart from the 167 °F case. Once again, the trends for the stress in the patch, both cracked and uncracked, as a function of operating temperature are reversed. The results from the proposed equations are considered superior in terms of both absolute accuracy and in capturing the physical trends expected from structure subjected to temperature change.

Also from Table 4, the values of  $\sigma_o$ ,  $\gamma_{max}$  and  $K_{inf}$  do not change significantly.

## 5. Conclusion and Recommendations

Improved methods for analysing load attraction and thermal effects in bonded composite repair design have been evaluated by way of example calculations. The proposed equations are considered to be more accurate than those currently in use, but remain conservative.

The use of the proposed equations in the next update to Reference 6 is strongly recommended.

## 6. Acknowledgements

The authors greatly appreciate the input and advice from Drs C. Wang, A. Baker and A. Rider.

## 7. References

1. Baker, A.A., and Jones, R., (eds), 1988, Bonded Repair of Aircraft Structures, Martinus Nijhoff.
2. Baker, A.A., October 3-5, 1994, Bonded Composite Repair of Metallic Aircraft Components - Overview of Australian Activities, AGARD Specialists Meeting on Composite Repair of Military Aircraft Structures, Seville.

3. Baker, A.A., et.al., 1999, Repair Substantiation for a Bonded Composite Repair to F-111 Lower Wing Skin, *Journal of Applied Composite Materials* 6:251-667, Kluwer Academic Publishers, Netherlands.
4. Boykett, R., and Walker, K., 1996, F-111C Lower Wing Skin Bonded Composite Repair Substantiation Testing", DSTO-TR-0480.
5. Anon, 4 Sept., 1995, Composite Materials and Adhesive Bonded Repairs, Standard DEF(AUST)9005 Issue A (Draft) replacing RAAF STD (ENG) C5033.
6. Anon, Draft 2002, Composite Materials and Adhesive Bonded Repairs - Engineering and Design Procedures", RAAF AAP 7021.016-1.
7. Hart-Smith, L.J., 1973, Adhesive Bonded Double Lap Joints, NASA-CR-012235, California.
8. Rose, L.R.F., Wang, C.H., 2002, Analytical Methods for Designing Composite Repairs (Chapter 7), *Advances in the Bonded Composite Repair of Metallic Aircraft Structure*, Vol. 1, Baker, A.A. et.al., Elsevier, Oxford, pp 137-175.
9. Wang, C.H., et.al., 2000, Thermal Stresses in a Plate with a Circular Reinforcement, *International Journal of Solids and Structures*, Vol 37, 4577-4599.
10. Anon, 3 May 1994, Repair Design Package for A8-145, ASI/4080/14/A8-12 Pt 4(34).

## Appendix A Calculations for F-111 Repair Example

This appendix details the mathematical workings for the F-111 repair design example application. Both the current analytic formulae and the proposed analytic formulae have been used independently, as a means of quantifying the effect of implementing the improved methods.

The calculations have been performed using MATHCAD worksheets. Therefore, each of the repair examples are set up in a sequential fashion, much like the way specified in the current RAAF standard for bonded repair design. Worksheets have been provided for both the current methodology and the improved methodology, for the following temperature conditions:

- -40 F
- 75 F (Ambient)
- 167 F

## A.1. Current Method – Operating Temperature = -40 °F

### Single-Sided Fully-Supported Bonded Repair Evaluation of Current Equations

#### ADHESIVE PROPERTIES

#### FM73 FILM ADHESIVE (Minus 40 deg F CONDITIONS)

ADHESIVE THICKNESS      ADHESIVE SHEAR MODULUS      ADHESIVE SHEAR STRESS

$$\eta := 0.013 \text{ inch} \quad G := 115306 \text{ psi} \quad \tau_p := 8230 \text{ psi}$$

ADHESIVE STRAINS      ELASTIC       $\gamma_e := 0.0723$

$$\text{PLASTIC} \quad \gamma_p := 0.1192$$

CURE TEMPERATURE       $T_{cure} := 180 \text{ F}$

#### 2024-T851 ALUMINIUM SKIN PROPERTIES

SKIN THICKNESS       $t_i := 0.14 \text{ in}$

YIELD STRESS       $\sigma_{yi} := 59 \cdot 10^3 \text{ psi}$

ULTIMATE STRESS       $\sigma_{ui} := 65 \cdot 10^3 \text{ psi}$

FRACTURE TOUGHNESS       $K_c := 42000 \text{ psi rt in}$

YOUNGS MODULUS       $E_i := 10.5 \cdot 10^6 \text{ psi}$

POISSON'S RATIO       $\nu := 0.31$

THERMAL EXPANSN COEFFICIENT       $\alpha_i := 12.6 \cdot 10^{-6} \text{ in/in/F}$        $\alpha_{ieff} := \alpha_i \cdot \frac{(\nu + 1)}{2}$

#### BORON EPOXY 5521/4 PROPERTIES

##### BORON EPOXY PRE-PREG

ULTIMATE LONGITUDINAL STRAIN       $\varepsilon_o := 0.00655 \text{ in/in}$

YOUNGS MODULUS       $E_o := 30 \cdot 10^6 \text{ psi}$

THERMAL EXPANSN COEFFICIENT       $\alpha_o := 2.3 \cdot 10^{-6} \text{ in/in/F}$

THICKNESS       $t_o := 0.052 \text{ in}$

DESIGN OPERATING TEMPERATURE       $T_{op} := -40 \text{ deg F}$        $RT := 75 \text{ deg F}$

**EVALUATION OF STRUCTURAL INTEGRITY OF REPAIRED STRUCTURE****CALCULATE THE STRESS APPLIED**

$$\text{Input remote applied stress} \quad \sigma_{star} := 37100 \quad \text{psi}$$

**CHECK THE STRUCTURAL INTEGRITY OF THE STRUCTURE AT THE EDGE OF THE PATCH**

$$\text{Load Attraction Factor} \quad \Omega_L := 1.2$$

Stress at the edge of the patch

$$\sigma_{2star} := \Omega_L \cdot \sigma_{star} + E_i \left[ (\alpha_o - \alpha_{i\text{eff}}) \cdot (RT - T_{cure}) + (\alpha_o - \alpha_i) \cdot (T_{op} - RT) \right]$$

$$\sigma_{2star} = 6.352 \times 10^4$$

**CALCULATE THE STRESS UNDER THE REPAIR**

$$\sigma_o := \frac{\sigma_{2star} \cdot (E_i \cdot t_i)}{E_o \cdot t_o + E_i \cdot t_i} \quad \sigma_o = 3.082 \times 10^4$$

**CHECK THE ADHESIVE SHEAR STRAIN**

$$\lambda := \sqrt{\left(\frac{G}{\eta}\right) \left[ \left(\frac{1}{E_i \cdot t_i}\right) + \left(\frac{1}{E_o \cdot t_o}\right) \right]} \quad \lambda = 3.423$$

$$\text{Elastic Shear Strain} \quad \gamma_{maxE} := \sigma_o \cdot t_i \cdot \frac{\lambda}{G} \quad \gamma_{maxE} = 0.128$$

$$\text{Plastic Shear Strain} \quad \gamma_{maxP} := \left( \frac{\tau_p}{2 \cdot G} \right) \left[ 1 + \left( \sigma_o \cdot t_i \cdot \frac{\lambda}{\tau_p} \right)^2 \right] \quad \gamma_{maxP} = 0.151$$

$$\gamma_{max} := \text{if}(\gamma_{maxE} > \gamma_e, \gamma_{maxP}, \gamma_{maxE})$$

$$\text{Maximum Shear Strain in Adhesive} \quad \gamma_{max} = 0.151$$

**CALCULATE THE STRESS INTENSITY IN A REPAIRED METALLIC STRUCTURE**

$$K_{injE} := \sigma_o \cdot \sqrt{\frac{(E_i \cdot t_i \cdot \lambda \cdot \eta)}{G}} \quad K_{injE} = 2.321 \times 10^4$$

$$K_{injP} := \sqrt{\frac{E_i \cdot \eta}{G}} \left[ \sigma_o \cdot \tau_p \left[ 1 + \left( \frac{\sigma_o \cdot \lambda \cdot t_i}{\tau_p} \right)^2 \right] - \frac{\tau_p^2}{\lambda \cdot t_i \cdot 3} \left[ 1 + 2 \cdot \left( \frac{\sigma_o \cdot \lambda \cdot t_i}{\tau_p} \right)^3 \right] \right]$$

$$K_{injP} = 2.381 \times 10^4$$

$$K_{inj} := \text{if}(\gamma_{maxE} > \gamma_e, K_{injP}, K_{injE})$$

$$\text{Repaired SIF} \quad K_{inj} = 2.381 \times 10^4 \quad \text{psi rt in}$$

CHECK THE STRUCTURAL INTEGRITY OF THE PATCH

$$\sigma_p := \frac{(\sigma_{2star} \cdot t_i)}{t_o} \quad \sigma_{2star} = 6.352 \times 10^4$$
$$\sigma_p = 1.71 \times 10^5 \quad \text{psi}$$

IN THE ABSENCE OF THE CRACK

$$\sigma_p := \frac{(E_o \cdot t_i \cdot \sigma_{2star})}{(E_o \cdot t_o + E_i \cdot t_i)}$$

$$\sigma_p = 8.805 \times 10^4$$

## A.2. Current Method – Operating Temperature = 75 °F

### Single-Sided Fully-Supported Bonded Repair Evaluation of Current Equations

#### ADHESIVE PROPERTIES

##### FM73 FILM ADHESIVE (ROOM TEMP CONDITIONS)

ADHESIVE THICKNESS      ADHESIVE SHEAR MODULUS      ADHESIVE SHEAR STRESS

$$\eta := 0.013 \quad \text{inch} \quad G := 73323 \quad \text{psi} \quad \tau_p := 6052 \quad \text{psi}$$

ADHESIVE STRAINS      ELASTIC       $\gamma_e := 0.0804$

$$\text{PLASTIC} \quad \gamma_p := 0.497$$

CURE TEMPERATURE       $T_{cure} := 180 \text{ } ^\circ\text{F}$

#### 2024-T851 ALUMINIUM SKIN PROPERTIES

SKIN THICKNESS       $t_i := 0.14 \text{ } \text{in}$

$$\text{YIELD STRESS} \quad \sigma_{yi} := 59 \cdot 10^3 \quad \text{psi}$$

$$\text{ULTIMATE STRESS} \quad \sigma_{ui} := 65 \cdot 10^3 \quad \text{psi}$$

FRACTURE TOUGHNESS       $K_c := 42000 \text{ } \text{psi rt in}$

$$\text{YOUNGS MODULUS} \quad E_i := 10.5 \cdot 10^6 \quad \text{psi}$$

POISSON'S RATIO       $\nu := 0.31$

$$\text{THERMAL EXPANSN COEFFICIENT} \quad \alpha_i := 12.6 \cdot 10^{-6} \text{ in/in/F} \quad \alpha_{i\text{eff}} := \alpha_i \cdot \frac{(\nu + 1)}{2}$$

#### BORON EPOXY 5521/4 PROPERTIES

BORON EPOXY PRE-PREG

ULTIMATE LONGITUDINAL STRAIN       $\varepsilon_o := 0.00655 \text{ } \text{in/in}$

$$\text{YOUNGS MODULUS} \quad E_o := 30 \cdot 10^6 \quad \text{psi}$$

$$\text{THERMAL EXPANSN COEFFICIENT} \quad \alpha_o := 2.3 \cdot 10^{-6} \text{ in/in/F}$$

THICKNESS       $t_o := 0.052 \text{ } \text{in}$

$$\text{DESIGN OPERATING TEMPERATURE} \quad T_{op} := 75 \text{ } \text{deg F} \quad RT := 75 \text{ } \text{deg F}$$

**EVALUATION OF STRUCTURAL INTEGRITY OF REPAIRED STRUCTURE****CALCULATE THE STRESS APPLIED**

$$\text{Input remote applied stress} \quad \sigma_{star} := 37100 \quad \text{psi}$$

**CHECK THE STRUCTURAL INTEGRITY OF THE STRUCTURE AT THE EDGE OF THE PATCH**

$$\text{Load Attraction Factor} \quad \Omega_L := 1.2$$

Stress at the edge of the patch

$$\sigma_{2star} := \Omega_L \cdot \sigma_{star} + E_i \left[ (\alpha_o - \alpha_{i\text{eff}}) \cdot (RT - T_{cure}) + (\alpha_o - \alpha_i) \cdot (T_{op} - RT) \right]$$

$$\sigma_{2star} = 5.108 \times 10^4$$

**CALCULATE THE STRESS UNDER THE REPAIR**

$$\sigma_o := \frac{\sigma_{2star} \cdot (E_i \cdot t_i)}{E_o \cdot t_o + E_i \cdot t_i} \quad \sigma_o = 2.478 \times 10^4$$

**STEP3(d): CHECK THE ADHESIVE SHEAR STRAIN**

$$\lambda := \sqrt{\left(\frac{G}{\eta}\right)} \left[ \left(\frac{1}{E_i \cdot t_i}\right) + \left(\frac{1}{E_o \cdot t_o}\right) \right] \quad \lambda = 2.73$$

$$\text{Elastic Shear Strain} \quad \gamma_{maxE} := \sigma_o \cdot t_i \cdot \frac{\lambda}{G} \quad \gamma_{maxE} = 0.129$$

$$\text{Plastic Shear Strain} \quad \gamma_{maxP} := \left( \frac{\tau_p}{2 \cdot G} \right) \left[ 1 + \left( \frac{\sigma_o \cdot t_i \cdot \lambda}{\tau_p} \right)^2 \right] \quad \gamma_{maxP} = 0.142$$

$$\gamma_{max} := if(\gamma_{maxE} > \gamma_e, \gamma_{maxP}, \gamma_{maxE})$$

$$\text{Maximum Shear Strain in Adhesive} \quad \gamma_{max} = 0.142$$

**CALCULATE THE STRESS INTENSITY IN A REPAIRED METALLIC STRUCTURE**

$$K_{infe} := \sigma_o \cdot \sqrt{\frac{(E_i \cdot t_i \cdot \lambda \cdot \eta)}{G}} \quad K_{infe} = 2.09 \times 10^4$$

$$K_{infP} := \sqrt{\frac{E_i \cdot \eta}{G}} \left[ \sigma_o \cdot \tau_p \left[ 1 + \left( \frac{\sigma_o \cdot \lambda \cdot t_i}{\tau_p} \right)^2 \right] - \frac{\tau_p^2}{\lambda \cdot t_i \cdot 3} \left[ 1 + 2 \left( \frac{\sigma_o \cdot \lambda \cdot t_i}{\tau_p} \right)^3 \right] \right]$$

$$K_{infP} = 2.116 \times 10^4$$

$$K_{inf} := if(\gamma_{maxE} > \gamma_e, K_{infP}, K_{infe})$$

$$\text{Repaired SIF} \quad K_{inf} = 2.116 \times 10^4 \quad \text{psi rt in}$$

*CHECK THE STRUCTURAL INTEGRITY OF THE PATCH*

$$\sigma_p := \frac{(\sigma_{2star} \cdot t_i)}{t_o}$$

$$\sigma_p = 1.375 \times 10^5 \text{ psi}$$

*IN THE ABSENCE OF THE CRACK*

$$\sigma_p := \frac{(E_o \cdot t_i \cdot \sigma_{2star})}{(E_o \cdot t_o + E_i \cdot t_i)}$$

$$\sigma_p = 7.081 \times 10^4$$

### A.3. Current Method – Operating Temperature = 167 °F

#### Single-Sided Fully-Supported Bonded Repair Evaluation of Current Equations

##### ADHESIVE PROPERTIES FM73 FILM ADHESIVE (167 deg F CONDITIONS)

ADHESIVE THICKNESS      ADHESIVE SHEAR MODULUS      ADHESIVE SHEAR STRESS

$$\eta := 0.013 \quad \text{inch} \quad G := 13522 \quad \text{psi} \quad \tau_p := 3534 \quad \text{psi}$$

ADHESIVE STRAINS      ELASTIC       $\gamma_e := 0.5896$

$$\text{PLASTIC} \quad \gamma_p := 0.2380$$

CURE TEMPERATURE       $T_{cure} := 180 \text{ } ^\circ\text{F}$

##### 2024-T851 ALUMINIUM SKIN PROPERTIES

SKIN THICKNESS       $t_i := 0.14 \text{ } \text{in}$

$$\text{YIELD STRESS} \quad \sigma_{yi} := 59 \cdot 10^3 \quad \text{psi}$$

$$\text{ULTIMATE STRESS} \quad \sigma_{ui} := 65 \cdot 10^3 \quad \text{psi}$$

FRACTURE TOUGHNESS       $K_c := 42000 \text{ } \text{psi rt in}$

$$\text{YOUNGS MODULUS} \quad E_i := 10.5 \cdot 10^6 \quad \text{psi}$$

POISSON'S RATIO       $\nu := 0.31$

$$\text{THERMAL EXPANSN COEFFICIENT} \quad \alpha_i := 12.6 \cdot 10^{-6} \text{ in/in/F} \quad \alpha_{i\text{eff}} := \alpha_i \cdot \frac{(\nu + 1)}{2}$$

##### BORON EPOXY 5521/4 PROPERTIES

BORON EPOXY PRE-PREG

ULTIMATE LONGITUDINAL STRAIN       $\varepsilon_o := 0.00655 \text{ } \text{in/in}$

$$\text{YOUNGS MODULUS} \quad E_o := 30 \cdot 10^6 \quad \text{psi}$$

$$\text{THERMAL EXPANSN COEFFICIENT} \quad \alpha_o := 2.3 \cdot 10^{-6} \text{ in/in/F}$$

$$\text{THICKNESS} \quad t_o := 0.052 \text{ } \text{in}$$

$$\text{DESIGN OPERATING TEMPERATURE} \quad T_{op} := 167 \text{ } \text{deg F} \quad RT := 75 \text{ } \text{deg F}$$

**EVALUATION OF STRUCTURAL INTEGRITY OF REPAIRED STRUCTURE****CALCULATE THE STRESS APPLIED**

Input remote applied stress       $\sigma_{star} := 37100 \text{ psi}$

CHECK THE STRUCTURAL INTEGRITY OF THE STRUCTURE AT THE EDGE OF THE PATCH

Load Attraction Factor       $\Omega_L := 1.2$

Stress at the edge of the patch

$$\sigma_{2star} := \Omega_L \cdot \sigma_{star} + E_i \left[ (\alpha_o - \alpha_{i\text{eff}}) \cdot (RT - T_{cure}) + (\alpha_o - \alpha_i) \cdot (T_{op} - RT) \right]$$

$$\sigma_{2star} = 4.113 \times 10^4$$

**CALCULATE THE STRESS UNDER THE REPAIR**

$$\sigma_o := \frac{\sigma_{2star} \cdot (E_i \cdot t_i)}{E_o \cdot t_o + E_i \cdot t_i} \quad \sigma_o = 1.996 \times 10^4$$

CHECK THE ADHESIVE SHEAR STRAIN

$$\lambda := \sqrt{\left(\frac{G}{\eta}\right) \left[ \left(\frac{1}{E_i \cdot t_i}\right) + \left(\frac{1}{E_o \cdot t_o}\right) \right]} \quad \lambda = 1.172$$

$$\text{Elastic Shear Strain} \quad \gamma_{maxE} := \sigma_o \cdot t_i \cdot \frac{\lambda}{G} \quad \gamma_{maxE} = 0.242$$

$$\text{Plastic Shear Strain} \quad \gamma_{maxP} := \left( \frac{\tau_p}{2 \cdot G} \right) \left[ 1 + \left( \sigma_o \cdot t_i \cdot \frac{\lambda}{\tau_p} \right)^2 \right] \quad \gamma_{maxP} = 0.243$$

$$\gamma_{max} := \text{if}(\gamma_{maxE} > \gamma_e, \gamma_{maxP}, \gamma_{maxE})$$

$$\text{Maximum Shear Strain in Adhesive} \quad \gamma_{max} = 0.242$$

**CALCULATE THE STRESS INTENSITY IN A REPAIRED METALLIC STRUCTURE**

$$K_{injE} := \sigma_o \cdot \sqrt{\frac{(E_i \cdot t_i \cdot \lambda \cdot \eta)}{G}} \quad K_{injE} = 2.569 \times 10^4$$

$$K_{injP} := \sqrt{\frac{E_i \cdot \eta}{G} \cdot \left[ \sigma_o \cdot \tau_p \cdot \left[ 1 + \left( \frac{\sigma_o \cdot \lambda \cdot t_i}{\tau_p} \right)^2 \right] - \frac{\tau_p^2}{\lambda \cdot t_i \cdot 3} \cdot \left[ 1 + 2 \cdot \left( \frac{\sigma_o \cdot \lambda \cdot t_i}{\tau_p} \right)^3 \right] \right]}$$

$$K_{injP} = 2.568 \times 10^4$$

$$K_{inj} := \text{if}(\gamma_{maxE} > \gamma_e, K_{injP}, K_{injE})$$

$$\text{Repaired SIF} \quad K_{inj} = 2.569 \times 10^4 \quad \text{psi rt in}$$

*CHECK THE STRUCTURAL INTEGRITY OF THE PATCH*

$$\sigma_p := \frac{(\sigma_{2star} \cdot t_i)}{t_o}$$

$$\sigma_p = 1.107 \times 10^5 \text{ psi}$$

*IN THE ABSENCE OF THE CRACK*

$$\sigma_p := \frac{(E_o \cdot t_i \cdot \sigma_{2star})}{(E_o \cdot t_o + E_i \cdot t_i)}$$

$$\sigma_p = 5.702 \times 10^4$$

## A.4. Improved Method – Operating Temperature = -40 °F

### Single-Sided Fully-Supported Bonded Repair Evaluation of Proposed Equations

#### ADHESIVE PROPERTIES

##### FM73 FILM ADHESIVE (Minus 40 deg F CONDITIONS)

ADHESIVE THICKNESS      ADHESIVE SHEAR MODULUS      ADHESIVE SHEAR STRESS

$$\eta := 0.013 \text{ inch} \quad G := 115306 \text{ psi} \quad \tau_p := 8230 \text{ psi}$$

YOUNG'S MODULUS:

$$\mu := 0.30$$

$$E_c := 2 \cdot G \cdot (1 + \mu) \quad E_c = 2.998 \times 10^5 \text{ psi}$$

ADHESIVE STRAINS      ELASTIC       $\gamma_e := 0.0723$

PLASTIC       $\gamma_p := 0.1192$

CURE TEMPERATURE       $T_{cure} := 180 \text{ F}$

#### 2024-T851 ALUMINIUM SKIN PROPERTIES

SKIN THICKNESS       $t_i := 0.14 \text{ in}$

YIELD STRESS       $\sigma_{yi} := 59 \cdot 10^3 \text{ psi}$

ULTIMATE STRESS       $\sigma_{ui} := 65 \cdot 10^3 \text{ psi}$

FRACTURE TOUGHNESS       $K_c := 42000 \text{ psi rt in}$

YOUNG'S MODULUS       $E_i := 10.5 \cdot 10^6 \text{ psi}$

POISSON'S RATIO       $\nu := 0.31$

THERMAL EXPANSN COEFFICIENT       $\alpha_i := 12.6 \cdot 10^{-6} \text{ in/in/F}$        $\alpha_{ieff} := \alpha_i - \frac{(\nu + 1)}{2}$

#### BORON EPOXY 5521/4 PROPERTIES

BORON EPOXY PRE-PREG

ULTIMATE LONGITUDINAL STRAIN       $\epsilon_0 := 0.00655 \text{ in/in}$

YOUNG'S MODULUS       $E_0 := 30 \cdot 10^6 \text{ psi}$

THERMAL EXPANSN COEFFICIENT       $\alpha_0 := 2.3 \cdot 10^{-6} \text{ in/in/F}$

POISSON'S RATIO       $\nu_{210} := 0.019$        $\nu_{120} := 0.21$

THICKNESS       $t_0 := 0.052 \text{ in}$

DESIGN OPERATING TEMPERATURE       $T_{op} := -40 \text{ deg F}$        $RT := 75 \text{ deg F}$

**EVALUATION OF STRUCTURAL INTEGRITY OF REPAIRED STRUCTURE****CALCULATE THE STRESS APPLIED**

Input remote applied stress       $\sigma_{\text{star}} := 37100 \text{ psi}$

**CHECK THE STRUCTURAL INTEGRITY OF THE STRUCTURE AT THE EDGE OF THE PATCH**

$$\text{Stiffness Ratio} \quad S := \frac{(E_o \cdot t_o)}{(E_i \cdot t_i)} \quad S = 1.061$$

$$\text{Load Attraction Factor} \quad \Omega_L := 1.2$$

Stress at the edge of the patch due to applied load

$$\sigma_{2\text{starapplied}} := \Omega_L \cdot \sigma_{\text{star}}$$

$$\sigma_{2\text{starapplied}} = 4.452 \times 10^4$$

Stress at the edge of the patch due to heating during cure

$$\sigma_{2\text{starheating}} := -\alpha_i \cdot E_i \cdot \frac{(T_{\text{cure}} - RT)}{2} \quad \sigma_{2\text{starheating}} = -6.946 \times 10^3$$

Stress at the edge of the patch due to cooling to ambient during cure

$$\sigma_{2\text{starcooling}} := -\alpha_i \cdot E_i \cdot (RT - T_{\text{cure}}) \cdot \frac{\left[ 1 - v_{120} + S \cdot (1 - v) \cdot \frac{\alpha_o}{\alpha_i} \right]}{2 \cdot (1 - v_{120}) + (1 - v^2) \cdot S}$$

$$\sigma_{2\text{starcooling}} = 5.053 \times 10^3$$

Stress at the edge of the patch due to change to operating temperature

$$\sigma_{2\text{staroperating}} := \alpha_i \cdot E_i \cdot (T_{\text{op}} - RT) \cdot \frac{\left[ (1 - v) \cdot \left( 1 - \frac{\alpha_o}{\alpha_i} \right) \cdot S \right]}{2 \cdot (1 - v_{120}) + (1 - v^2) \cdot S}$$

$$\sigma_{2\text{staroperating}} = -3.587 \times 10^3$$

$$\sigma_{2\text{star}} := \sigma_{2\text{starapplied}} + \sigma_{2\text{starheating}} + \sigma_{2\text{starcooling}} + \sigma_{2\text{staroperating}}$$

$$\sigma_{2\text{star}} = 3.904 \times 10^4$$

**CALCULATE THE STRESS UNDER THE REPAIR**

Stress under patch due to applied load

$$\sigma_{\text{oapplied}} := \frac{(\Omega_L \cdot \sigma_{\text{star}})}{(1 + S)} \quad \sigma_{\text{oapplied}} = 2.16 \times 10^4$$

Stress under patch due to heating during cure

$$\sigma_{\text{oheating}} := -\alpha_i \cdot E_i \cdot \frac{(T_{\text{cure}} - RT)}{2} \quad \sigma_{\text{oheating}} = -6.946 \times 10^3$$

*Stress under patch due to cooling during cure*

$$\sigma_{\text{ocooling}} := -\alpha_i \cdot E_i \cdot (R\bar{T} - T_{\text{cure}}) \cdot \left[ \frac{\left[ 1 - v_{120} + \left( 1 - \frac{\alpha_o}{\alpha_i} \right) \cdot (1 + v) \cdot S \right]}{\left[ 2 \cdot (1 - v_{120}) + (1 - v)^2 \cdot S \right]} \right]$$

$$\sigma_{\text{ocooling}} = 1.054 \times 10^4$$

*Stress under patch due to change to operating temperature*

$$\sigma_{\text{ooperating}} := -\alpha_i \cdot E_i \cdot (T_{\text{op}} - R\bar{T}) \cdot \left[ \frac{\left[ (1 + v) \cdot \left( 1 - \frac{\alpha_o}{\alpha_i} \right) \cdot S \right]}{\left[ 2 \cdot (1 - v_{120}) + (1 - v)^2 \cdot S \right]} \right]$$

$$\sigma_{\text{ooperating}} = 6.809 \times 10^3$$

*Stress under patch (total)*

$$\sigma_o := \sigma_{\text{applied}} + \sigma_{\text{heating}} + \sigma_{\text{ocooling}} + \sigma_{\text{ooperating}}$$

$$\sigma_o = 3.2 \times 10^4$$

*CHECK THE ADHESIVE SHEAR STRAIN*

$$\lambda := \sqrt{\left( \frac{G}{\eta} \right) \cdot \left[ \left( \frac{1}{E_i \cdot t_i} \right) + \left( \frac{1}{E_o \cdot t_o} \right) \right]} \quad \lambda = 3.423$$

$$\text{Elastic Shear Strain} \quad \gamma_{\text{maxE}} := \sigma_o \cdot t_i \cdot \frac{\lambda}{G} \quad \gamma_{\text{maxE}} = 0.133$$

$$\text{Plastic Shear Strain} \quad \gamma_{\text{maxP}} := \left( \frac{\tau_p}{2 \cdot G} \right) \cdot \left[ 1 + \left( \sigma_o \cdot t_i \cdot \frac{\lambda}{\tau_p} \right)^2 \right] \quad \gamma_{\text{maxP}} = 0.16$$

$$\gamma_{\text{max}} := \text{if}(\gamma_{\text{maxE}} > \gamma_e, \gamma_{\text{maxP}}, \gamma_{\text{maxE}})$$

$$\text{Maximum Shear Strain in Adhesive} \quad \gamma_{\text{max}} = 0.16$$

*CALCULATE THE STRESS INTENSITY IN A REPAIRED METALLIC STRUCTURE*

$$K_{\text{infE}} := \sigma_o \cdot \sqrt{\frac{(E_i \cdot t_i \cdot \lambda \cdot \eta)}{G}} \quad K_{\text{infE}} = 2.41 \times 10^4$$

$$K_{\text{infP}} := \sqrt{\frac{E_i \cdot \eta}{G} \cdot \left[ \sigma_o \cdot \tau_p \cdot \left[ 1 + \left( \frac{\sigma_o \cdot \lambda \cdot t_i}{\tau_p} \right)^2 \right] - \frac{\tau_p^2}{\lambda \cdot t_i \cdot 3} \cdot \left[ 1 + 2 \cdot \left( \frac{\sigma_o \cdot \lambda \cdot t_i}{\tau_p} \right)^3 \right] \right]}$$

$$K_{\text{infP}} = 2.484 \times 10^4$$

$$K_{\text{inf}} := \text{if}(\gamma_{\text{maxE}} > \gamma_e, K_{\text{infP}}, K_{\text{infE}})$$

$$\text{Repaired SIF} \quad K_{\text{inf}} = 2.484 \times 10^4 \quad \text{psi rt in}$$

*CHECK THE STRUCTURAL INTEGRITY OF THE PATCH**Assuming a crack is present*

$$\sigma_p := \frac{(\sigma_{2\text{star}} \cdot t_i)}{t_0}$$

$$\sigma_p = 1.051 \times 10^5 \quad \text{psi}$$

*Assuming no crack**Stress in patch due to applied load*

$$\sigma_{\text{applied}} := \frac{\Omega_L \cdot \sigma_{\text{star}} \cdot S \cdot t_i}{(1 + S) \cdot t_0} \quad \sigma_{\text{applied}} = 6.171 \times 10^4$$

*Stress in patch due to heating during cure**There is no stress in the patch due to heating because it is free to expand*

$$\sigma_{\text{p heating}} := 0$$

*Stress in patch due to cooling during cure*

$$\sigma_{\text{p cooling}} := \alpha_i \cdot E_0 \cdot (RT - T_{\text{cure}}) \cdot \left[ \frac{\left( 1 + v - 2 \cdot \frac{\alpha_0}{\alpha_i} \right)}{\left[ 2 \cdot (1 - v_{120}) + (1 - v^2) \cdot S \right]} \right]$$

$$\sigma_{\text{p cooling}} = -1.477 \times 10^4$$

*Stress in patch due to change to operating temperature*

$$\sigma_{\text{p operating}} := \alpha_i \cdot E_0 \cdot (T_{\text{op}} - RT) \cdot \left[ \frac{2 \cdot \left( 1 - \frac{\alpha_0}{\alpha_i} \right)}{\left[ 2 \cdot (1 - v_{120}) + (1 - v^2) \cdot S \right]} \right]$$

$$\sigma_{\text{p operating}} = -2.799 \times 10^4$$

*Stress in patch (total)*

$$\sigma_p := \sigma_{\text{applied}} + \sigma_{\text{p heating}} + \sigma_{\text{p cooling}} + \sigma_{\text{p operating}}$$

$$\sigma_p = 1.895 \times 10^4$$

## A.5. Improved Method – Operating Temperature = 75 °F

### Single-Sided Fully-Supported Bonded Repair Evaluation of Proposed Equations

#### ADHESIVE PROPERTIES FM73 FILM ADHESIVE (ROOM TEMP CONDITIONS)

ADHESIVE THICKNESS      ADHESIVE SHEAR MODULUS      ADHESIVE SHEAR STRESS

$$\eta := 0.013 \quad \text{inch} \quad G := 73323 \quad \text{psi} \quad \tau_p := 6052 \quad \text{psi}$$

YOUNGS MODULUS:

$$\mu := 0.30$$

$$E_c := 2 \cdot G \cdot (1 + \mu) \quad E_c = 1.906 \times 10^5 \quad \text{psi}$$

ADHESIVE STRAINS      ELASTIC       $\gamma_c := 0.0804$

PLASTIC       $\gamma_p := 0.497$

CURE TEMPERATURE       $T_{\text{cure}} := 180 \quad \text{F}$

#### 2024-T851 ALUMINIUM SKIN PROPERTIES

SKIN THICKNESS       $t_i := 0.14 \quad \text{in}$

YIELD STRESS       $\sigma_{yi} := 59 \cdot 10^3 \quad \text{psi}$

ULTIMATE STRESS       $\sigma_{ui} := 65 \cdot 10^3 \quad \text{psi}$

FRACTURE TOUGHNESS       $K_c := 42000 \quad \text{psi rt in}$

YOUNGS MODULUS       $E_i := 10.5 \cdot 10^6 \quad \text{psi}$

POISSON'S RATIO       $\nu := 0.31$

THERMAL EXPANSN COEFFICIENT       $\alpha_i := 12.6 \cdot 10^{-6} \text{ in/in/F}$        $\alpha_{i\text{eff}} := \alpha_i \cdot \frac{(\nu + 1)}{2}$

#### BORON EPOXY 5521/4 PROPERTIES

BORON EPOXY PRE-PREG

ULTIMATE LONGITUDINAL STRAIN       $\epsilon_0 := 0.00655 \quad \text{in/in}$

YOUNGS MODULUS       $E_0 := 30 \cdot 10^6 \quad \text{psi}$

THERMAL EXPANSN COEFFICIENT       $\alpha_0 := 2.3 \cdot 10^{-6} \quad \text{in/in/F}$

POISSON'S RATIO       $\nu_{210} := 0.019 \quad \nu_{120} := 0.21$

THICKNESS       $t_0 := 0.052 \quad \text{in}$

DESIGN OPERATING TEMPERATURE       $T_{\text{op}} := 75 \quad \text{deg F}$        $RT := 75 \quad \text{deg F}$

**EVALUATION OF STRUCTURAL INTEGRITY OF REPAIRED STRUCTURE****CALCULATE THE STRESS APPLIED**

Input remote applied stress       $\sigma_{\text{star}} := 37100 \text{ psi}$

**CHECK THE STRUCTURAL INTEGRITY OF THE STRUCTURE AT THE EDGE OF THE PATCH**

$$\text{Stiffness Ratio} \quad S := \frac{(E_o \cdot t_o)}{(E_i \cdot t_i)} \quad S = 1.061$$

$$\text{Load Attraction Factor} \quad \Omega_L := 1.2$$

Stress at the edge of the patch due to applied load

$$\sigma_{2\text{starapplied}} := \Omega_L \cdot \sigma_{\text{star}}$$

$$\sigma_{2\text{starapplied}} = 4.452 \times 10^4$$

Stress at the edge of the patch due to heating during cure

$$\sigma_{2\text{starheating}} := -\alpha_i \cdot E_i \cdot \frac{(T_{\text{cure}} - RT)}{2} \quad \sigma_{2\text{starheating}} = -6.946 \times 10^3$$

Stress at the edge of the patch due to cooling to ambient during cure

$$\sigma_{2\text{starcooling}} := -\alpha_i \cdot E_i \cdot (RT - T_{\text{cure}}) \cdot \frac{\left[ 1 - v_{120} + S \cdot (1 - v) \cdot \frac{\alpha_o}{\alpha_i} \right]}{2 \cdot (1 - v_{120}) + (1 - v^2) \cdot S}$$

$$\sigma_{2\text{starcooling}} = 5.053 \times 10^3$$

Stress at the edge of the patch due to change to operating temperature

$$\sigma_{2\text{staroperating}} := \alpha_i \cdot E_i \cdot (T_{\text{op}} - RT) \cdot \frac{\left[ (1 - v) \cdot \left( 1 - \frac{\alpha_o}{\alpha_i} \right) S \right]}{2 \cdot (1 - v_{120}) + (1 - v^2) \cdot S}$$

$$\sigma_{2\text{staroperating}} = 0$$

$$\sigma_{2\text{star}} := \sigma_{2\text{starapplied}} + \sigma_{2\text{starheating}} + \sigma_{2\text{starcooling}} + \sigma_{2\text{staroperating}}$$

$$\sigma_{2\text{star}} = 4.263 \times 10^4$$

**CALCULATE THE STRESS UNDER THE REPAIR**

Stress under patch due to applied load

$$\sigma_{\text{oapplied}} := \frac{(\Omega_L \cdot \sigma_{\text{star}})}{(1 + S)} \quad \sigma_{\text{oapplied}} = 2.16 \times 10^4$$

Stress under patch due to heating during cure

$$\sigma_{\text{oheating}} := -\alpha_i \cdot E_i \cdot \frac{(T_{\text{cure}} - RT)}{2} \quad \sigma_{\text{oheating}} = -6.946 \times 10^3$$

*Stress under patch due to cooling during cure*

$$\sigma_{\text{ocooling}} := -\alpha_i \cdot E_i \cdot (RT - T_{\text{cure}}) \cdot \left[ \frac{\left[ 1 - v_{120} + \left( 1 - \frac{\alpha_o}{\alpha_i} \right) \cdot (1 + v) \cdot S \right]}{\left[ 2 \cdot (1 - v_{120}) + (1 - v)^2 \cdot S \right]} \right]$$

$$\sigma_{\text{ocooling}} = 1.054 \times 10^4$$

*Stress under patch due to change to operating temperature*

$$\sigma_{\text{ooperating}} := -\alpha_i \cdot E_i \cdot (T_{\text{op}} - RT) \cdot \left[ \frac{\left[ (1 + v) \cdot \left( 1 - \frac{\alpha_o}{\alpha_i} \right) \cdot S \right]}{\left[ 2 \cdot (1 - v_{120}) + (1 - v)^2 \cdot S \right]} \right]$$

$$\sigma_{\text{ooperating}} = 0$$

*Stress under patch (total)*

$$\sigma_o := \sigma_{\text{applied}} + \sigma_{\text{heating}} + \sigma_{\text{ocooling}} + \sigma_{\text{ooperating}}$$

$$\sigma_o = 2.519 \times 10^4$$

*CHECK THE ADHESIVE SHEAR STRAIN*

$$\lambda := \sqrt{\left(\frac{G}{\eta}\right)} \cdot \left[ \left( \frac{1}{E_i \cdot t_i} \right) + \left( \frac{1}{E_o \cdot t_o} \right) \right] \quad \lambda = 2.73$$

$$\text{Elastic Shear Strain} \quad \gamma_{\text{maxE}} := \sigma_o \cdot t_i \cdot \frac{\lambda}{G} \quad \gamma_{\text{maxE}} = 0.131$$

$$\text{Plastic Shear Strain} \quad \gamma_{\text{maxP}} := \left( \frac{\tau_p}{2 \cdot G} \right) \cdot \left[ 1 + \left( \sigma_o \cdot t_i \cdot \frac{\lambda}{\tau_p} \right)^2 \right] \quad \gamma_{\text{maxP}} = 0.146$$

$$\gamma_{\text{max}} := \text{if}(\gamma_{\text{maxE}} > \gamma_e, \gamma_{\text{maxP}}, \gamma_{\text{maxE}})$$

$$\text{Maximum Shear Strain in Adhesive} \quad \gamma_{\text{max}} = 0.146$$

*CALCULATE THE STRESS INTENSITY IN A REPAIRED METALLIC STRUCTURE*

$$K_{\text{inffE}} := \sigma_o \cdot \sqrt{\left( \frac{(E_i \cdot t_i \cdot \lambda \cdot \eta)}{G} \right)} \quad K_{\text{inffE}} = 2.125 \times 10^4$$

$$K_{\text{infp}} := \sqrt{\frac{E_i \cdot \eta}{G}} \cdot \left[ \sigma_o \cdot \tau_p \cdot \left[ 1 + \left( \frac{\sigma_o \cdot \lambda \cdot t_i}{\tau_p} \right)^2 \right] - \frac{\tau_p^2}{\lambda \cdot t_i \cdot 3} \cdot \left[ 1 + 2 \cdot \left( \frac{\sigma_o \cdot \lambda \cdot t_i}{\tau_p} \right)^3 \right] \right]$$

$$K_{\text{infp}} = 2.154 \times 10^4$$

$$K_{\text{inf}} := \text{if}(\gamma_{\text{maxE}} > \gamma_e, K_{\text{infp}}, K_{\text{inffE}})$$

$$\text{Repaired SIF} \quad K_{\text{inf}} = 2.154 \times 10^4 \quad \text{psi rt in}$$

*CHECK THE STRUCTURAL INTEGRITY OF THE PATCH**Assuming a crack is present*

$$\sigma_p := \frac{(\sigma_{2\text{star}} \cdot t_i)}{t_0}$$

$$\sigma_p = 1.148 \times 10^5 \text{ psi}$$

*Assuming no crack**Stress in patch due to applied load*

$$\sigma_{\text{papplied}} := \frac{\Omega_L \cdot \sigma_{\text{star}} \cdot S \cdot t_i}{(1 + S) \cdot t_0} \quad \sigma_{\text{papplied}} = 6.171 \times 10^4$$

*Stress in patch due to heating during cure**There is no stress in the patch due to heating because it is free to expand*

$$\sigma_{\text{p heating}} := 0$$

*Stress in patch due to cooling during cure*

$$\sigma_{\text{p cooling}} := \alpha_i \cdot E_0 \cdot (RT - T_{\text{cure}}) \cdot \left[ \frac{\left( 1 + v - 2 \cdot \frac{\alpha_0}{\alpha_i} \right)}{\left[ 2 \cdot (1 - v_{120}) + (1 - v^2) \cdot S \right]} \right]$$

$$\sigma_{\text{p cooling}} = -1.477 \times 10^4$$

*Stress in patch due to change to operating temperature*

$$\sigma_{\text{p operating}} := \alpha_i \cdot E_0 \cdot (T_{\text{op}} - RT) \cdot \left[ \frac{2 \cdot \left( 1 - \frac{\alpha_0}{\alpha_i} \right)}{\left[ 2 \cdot (1 - v_{120}) + (1 - v^2) \cdot S \right]} \right]$$

$$\sigma_{\text{p operating}} = 0$$

*Stress in patch (total)*

$$\sigma_p := \sigma_{\text{papplied}} + \sigma_{\text{p heating}} + \sigma_{\text{p cooling}} + \sigma_{\text{p operating}}$$

$$\sigma_p = 4.694 \times 10^4$$

## A.6. Improved Method – Operating Temperature = 167 °F

### Single-Sided Fully-Supported Bonded Repair Evaluation of Proposed Equations

#### ADHESIVE PROPERTIES FM73 FILM ADHESIVE (167 deg F CONDITIONS)

ADHESIVE THICKNESS      ADHESIVE SHEAR MODULUS      ADHESIVE SHEAR STRESS

$$\eta := 0.013 \text{ inch} \quad G := 13522 \text{ psi} \quad \tau_p := 3534 \text{ psi}$$

YOUNGS MODULUS:

$$\mu := 0.30$$

$$E_c := 2 \cdot G \cdot (1 + \mu) \quad E_c = 3.516 \times 10^4 \text{ psi}$$

ADHESIVE STRAINS      ELASTIC       $\gamma_e := 0.5896$

PLASTIC       $\gamma_p := 0.2380$

CURE TEMPERATURE       $T_{cure} := 180 \text{ F}$

#### 2024-T851 ALUMINUM SKIN PROPERTIES

SKIN THICKNESS       $t_i := 0.14 \text{ in}$

YIELD STRESS       $\sigma_{yi} := 59 \cdot 10^3 \text{ psi}$

ULTIMATE STRESS       $\sigma_{ui} := 65 \cdot 10^3 \text{ psi}$

FRACTURE TOUGHNESS       $K_c := 42000 \text{ psi rt in}$

YOUNGS MODULUS       $E_i := 10.5 \cdot 10^6 \text{ psi}$

POISSON'S RATIO       $\nu := 0.31$

THERMAL EXPANSN COEFFICIENT       $\alpha_i := 12.6 \cdot 10^{-6} \text{ in/in/F}$        $\alpha_{ieff} := \alpha_i \cdot \frac{(\nu + 1)}{2}$

#### BORON EPOXY 5521/4 PROPERTIES

BORON EPOXY PRE-PREG

ULTIMATE LONGITUDINAL STRAIN       $\epsilon_0 := 0.00655 \text{ in/in}$

YOUNGS MODULUS       $E_0 := 30 \cdot 10^6 \text{ psi}$

THERMAL EXPANSN COEFFICIENT       $\alpha_0 := 2.3 \cdot 10^{-6} \text{ in/in/F}$

POISSON'S RATIO       $\nu_{210} := 0.019$        $\nu_{120} := 0.21$

THICKNESS       $t_0 := 0.052 \text{ in}$

DESIGN OPERATING TEMPERATURE       $T_{op} := 167 \text{ deg F}$        $RT := 75 \text{ deg F}$

**EVALUATION OF STRUCTURAL INTEGRITY OF REPAIRED STRUCTURE****CALCULATE THE STRESS APPLIED**

Input remote applied stress       $\sigma_{\text{star}} := 37100 \text{ psi}$

**CHECK THE STRUCTURAL INTEGRITY OF THE STRUCTURE AT THE EDGE OF THE PATCH**

$$\text{Stiffness Ratio} \quad S := \frac{(E_o \cdot t_o)}{(E_i \cdot t_i)} \quad S = 1.061$$

$$\text{Load Attraction Factor} \quad \Omega_L := 1.2$$

Stress at the edge of the patch due to applied load

$$\sigma_{2\text{starapplied}} := \Omega_L \cdot \sigma_{\text{star}}$$

$$\sigma_{2\text{starapplied}} = 4.452 \times 10^4$$

Stress at the edge of the patch due to heating during cure

$$\sigma_{2\text{starheating}} := -\alpha_i \cdot E_i \cdot \frac{(T_{\text{cure}} - RT)}{2} \quad \sigma_{2\text{starheating}} = -6.946 \times 10^3$$

Stress at the edge of the patch due to cooling to ambient during cure

$$\sigma_{2\text{starcooling}} := -\alpha_i \cdot E_i \cdot (RT - T_{\text{cure}}) \cdot \frac{\left[ 1 - v_{120} + S \cdot (1 - v) \cdot \frac{\alpha_o}{\alpha_i} \right]}{2 \cdot (1 - v_{120}) + (1 - v)^2 \cdot S}$$

$$\sigma_{2\text{starcooling}} = 5.053 \times 10^3$$

Stress at the edge of the patch due to change to operating temperature

$$\sigma_{2\text{staroperating}} := \alpha_i \cdot E_i \cdot (T_{\text{op}} - RT) \cdot \frac{\left[ (1 - v) \cdot \left( 1 - \frac{\alpha_o}{\alpha_i} \right) \cdot S \right]}{2 \cdot (1 - v_{120}) + (1 - v)^2 \cdot S}$$

$$\sigma_{2\text{staroperating}} = 2.869 \times 10^3$$

$$\sigma_{\text{2star}} := \sigma_{2\text{starapplied}} + \sigma_{2\text{starheating}} + \sigma_{2\text{starcooling}} + \sigma_{2\text{staroperating}}$$

$$\sigma_{\text{2star}} = 4.55 \times 10^4$$

**CALCULATE THE STRESS UNDER THE REPAIR**

Stress under patch due to applied load

$$\sigma_{\text{oapplied}} := \frac{(\Omega_L \cdot \sigma_{\text{star}})}{(1 + S)} \quad \sigma_{\text{oapplied}} = 2.16 \times 10^4$$

Stress under patch due to heating during cure

$$\sigma_{\text{oheating}} := -\alpha_i \cdot E_i \cdot \frac{(T_{\text{cure}} - RT)}{2} \quad \sigma_{\text{oheating}} = -6.946 \times 10^3$$

*Stress under patch due to cooling during cure*

$$\sigma_{\text{ocooling}} := -\alpha_i \cdot E_i \cdot (RT - T_{\text{cure}}) \cdot \left[ \frac{\left[ 1 - v_{120} + \left( 1 - \frac{\alpha_o}{\alpha_i} \right) \cdot (1 + v) \cdot S \right]}{\left[ 2 \cdot (1 - v_{120}) + (1 - v^2) \cdot S \right]} \right]$$

$$\sigma_{\text{ocooling}} = 1.054 \times 10^4$$

*Stress under patch due to change to operating temperature*

$$\sigma_{\text{ooperating}} := -\alpha_i \cdot E_i \cdot (T_{\text{op}} - RT) \cdot \left[ \frac{\left[ (1 + v) \cdot \left( 1 - \frac{\alpha_o}{\alpha_i} \right) \cdot S \right]}{\left[ 2 \cdot (1 - v_{120}) + (1 - v^2) \cdot S \right]} \right]$$

$$\sigma_{\text{ooperating}} = -5.447 \times 10^3$$

*Stress under patch (total)*

$$\sigma_o := \sigma_{\text{oapplied}} + \sigma_{\text{oheating}} + \sigma_{\text{ocooling}} + \sigma_{\text{ooperating}}$$

$$\sigma_o = 1.974 \times 10^4$$

*CHECK THE ADHESIVE SHEAR STRAIN*

$$\lambda := \sqrt{\left(\frac{G}{\eta}\right)} \cdot \left[ \left( \frac{1}{E_i \cdot t_i} \right) + \left( \frac{1}{E_o \cdot t_o} \right) \right] \quad \lambda = 1.172$$

$$\text{Elastic Shear Strain} \quad \gamma_{\text{maxE}} := \sigma_o \cdot t_i \cdot \frac{\lambda}{G} \quad \gamma_{\text{maxE}} = 0.24$$

$$\text{Plastic Shear Strain} \quad \gamma_{\text{maxP}} := \left( \frac{\tau_p}{2 \cdot G} \right) \cdot \left[ 1 + \left( \sigma_o \cdot t_i \cdot \frac{\lambda}{\tau_p} \right)^2 \right] \quad \gamma_{\text{maxP}} = 0.241$$

$$\gamma_{\text{max}} := \text{if}(\gamma_{\text{maxE}} > \gamma_e, \gamma_{\text{maxP}}, \gamma_{\text{maxE}})$$

$$\text{Maximum Shear Strain in Adhesive} \quad \gamma_{\text{max}} = 0.24$$

*CALCULATE THE STRESS INTENSITY IN A REPAIRED METALLIC STRUCTURE*

$$K_{\text{infE}} := \sigma_o \cdot \sqrt{\frac{(E_i \cdot t_i \cdot \lambda \cdot \eta)}{G}} \quad K_{\text{infE}} = 2.541 \times 10^4$$

$$K_{\text{infP}} := \sqrt{\frac{E_i \cdot \eta}{G}} \cdot \left[ \sigma_o \cdot \tau_p \cdot \left[ 1 + \left( \frac{\sigma_o \cdot \lambda \cdot t_i}{\tau_p} \right)^2 \right] - \frac{\tau_p^2}{\lambda \cdot t_i \cdot 3} \cdot \left[ 1 + 2 \cdot \left( \frac{\sigma_o \cdot \lambda \cdot t_i}{\tau_p} \right)^3 \right] \right]$$

$$K_{\text{infP}} = 2.541 \times 10^4$$

$$K_{\text{inf}} := \text{if}(\gamma_{\text{maxE}} > \gamma_e, K_{\text{infP}}, K_{\text{infE}})$$

$$\text{Repaired SIF} \quad K_{\text{inf}} = 2.541 \times 10^4 \quad \text{psi rt in}$$

*CHECK THE STRUCTURAL INTEGRITY OF THE PATCH**Assuming a crack is present*

$$\sigma_p := \frac{(\sigma_{2\text{star}} \cdot t_i)}{t_0}$$

$$\sigma_p = 1.225 \times 10^5 \text{ psi}$$

*Assuming no crack**Stress in patch due to applied load*

$$\sigma_{\text{papplied}} := \frac{\Omega_L \cdot \sigma_{\text{star}} \cdot S \cdot t_i}{(1 + S) \cdot t_0} \quad \sigma_{\text{papplied}} = 6.171 \times 10^4$$

*Stress in patch due to heating during cure**There is no stress in the patch due to heating because it is free to expand*

$$\sigma_{\text{p heating}} := 0$$

*Stress in patch due to cooling during cure*

$$\sigma_{\text{pcooling}} := \alpha_i \cdot E_0 \cdot (RT - T_{\text{cure}}) \cdot \left[ \frac{\left( 1 + \nu - 2 \cdot \frac{\alpha_0}{\alpha_i} \right)}{\left[ 2 \cdot (1 - \nu_{120}) + (1 - \nu^2) \cdot S \right]} \right]$$

$$\sigma_{\text{pcooling}} = -1.477 \times 10^4$$

*Stress in patch due to change to operating temperature*

$$\sigma_{\text{poperating}} := \alpha_i \cdot E_0 \cdot (T_{\text{op}} - RT) \cdot \left[ \frac{2 \left( 1 - \frac{\alpha_0}{\alpha_i} \right)}{\left[ 2 \cdot (1 - \nu_{120}) + (1 - \nu^2) \cdot S \right]} \right]$$

$$\sigma_{\text{poperating}} = 2.239 \times 10^4$$

*Stress in patch (total)*

$$\sigma_p := \sigma_{\text{papplied}} + \sigma_{\text{p heating}} + \sigma_{\text{pcooling}} + \sigma_{\text{poperating}}$$

$$\sigma_p = 6.933 \times 10^4$$

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Bonded Composite Repair Design

A. B. Harman and K. F. Walker

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Adhesive bonded repairs to aircraft involving metallic and composite structures have proven to be an effective, efficient means of repair and life extension. The simplified closed form equations used by the RAAF in an Engineering Standard (DEF(AUST)9005) have proven to be effective and conservative. Recent work, however, has identified improved equations to account for load attraction into the stiffened repaired area, and evaluate the thermally induced stresses in the repaired structure and the patch. The improved equations were compared with the current methods, using the repair to a 2024-T851 aluminium alloy F-111 lower wing skin with a boron epoxy repair patch, bonded with FM-73 film adhesive. The improved methods will reduce the unnecessary conservatism inherent in DEF(AUST)9005 and therefore allow some repairs to proceed where they may otherwise have been rejected. Repairs will also be designed to operate more efficiently.					